Modelling and Optimisation of Broadband Energy Harvesters Using Bistable Composites

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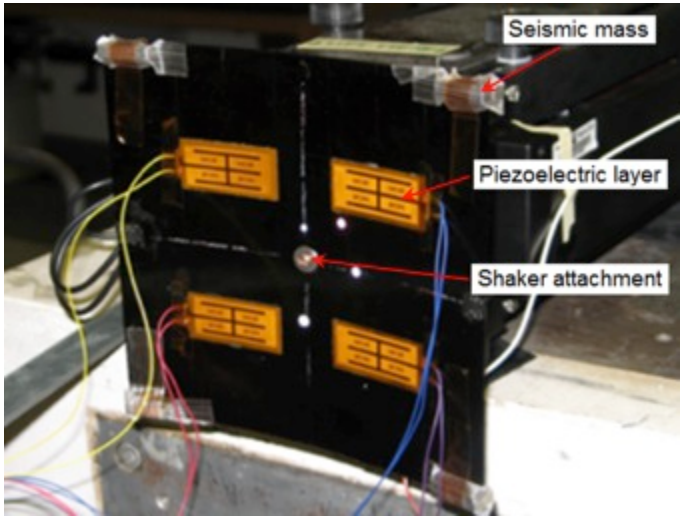
Broadband Energy Harvesters

- ~10-30mW power required for low power electronics (e.g. structural health monitoring)
- Most are tuned to vibrate at resonant frequencies → unsuitable for broadband ambient vibration

- Bistable systems have broadband energy harvesting characteristics due to nonlinearity (e.g. cantilever beam in a magnetic field)
- Bistable piezo-composites able to harvest energy from a wide range of frequencies
- EPSRC Project with Perpetuum, NPL



Arrieta, Inman APL 2010

Aim

To model and design vibration broadband energy harvesters using bistable composites.

- Introduction of bistable composites
- Modelling and investigation of dynamics of piezo-bistable composites
- Arbitary shape bistables with tailored mass distribution

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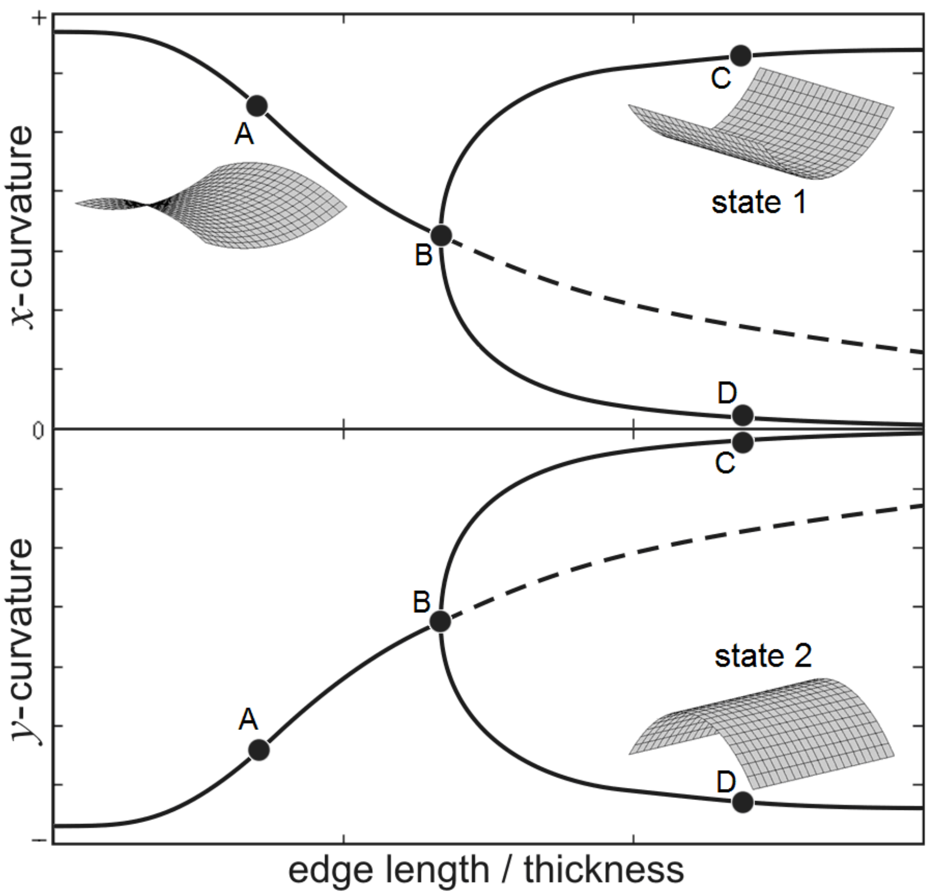
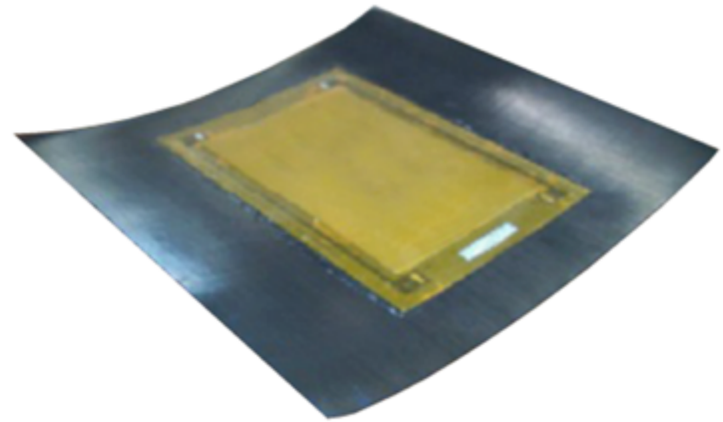
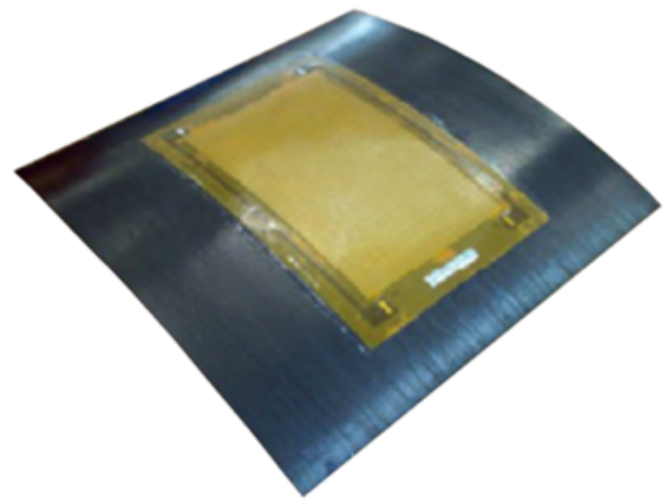
Bistable Composites – Asymmetric lay-up







Piezo-bistable composites based on statics.

Static Model

Existing modelling approach is based on minimization of total strain energy, W

$$\text{out-of-plane displacement} = \frac{1}{2}(ax^2 + by^2 + cxy)$$

W is the integral of strain energy density over the laminate volume,

a function of: out-of-plane (a-c), and in-plane shape coefficients ($d_{1\text{-}11}$),

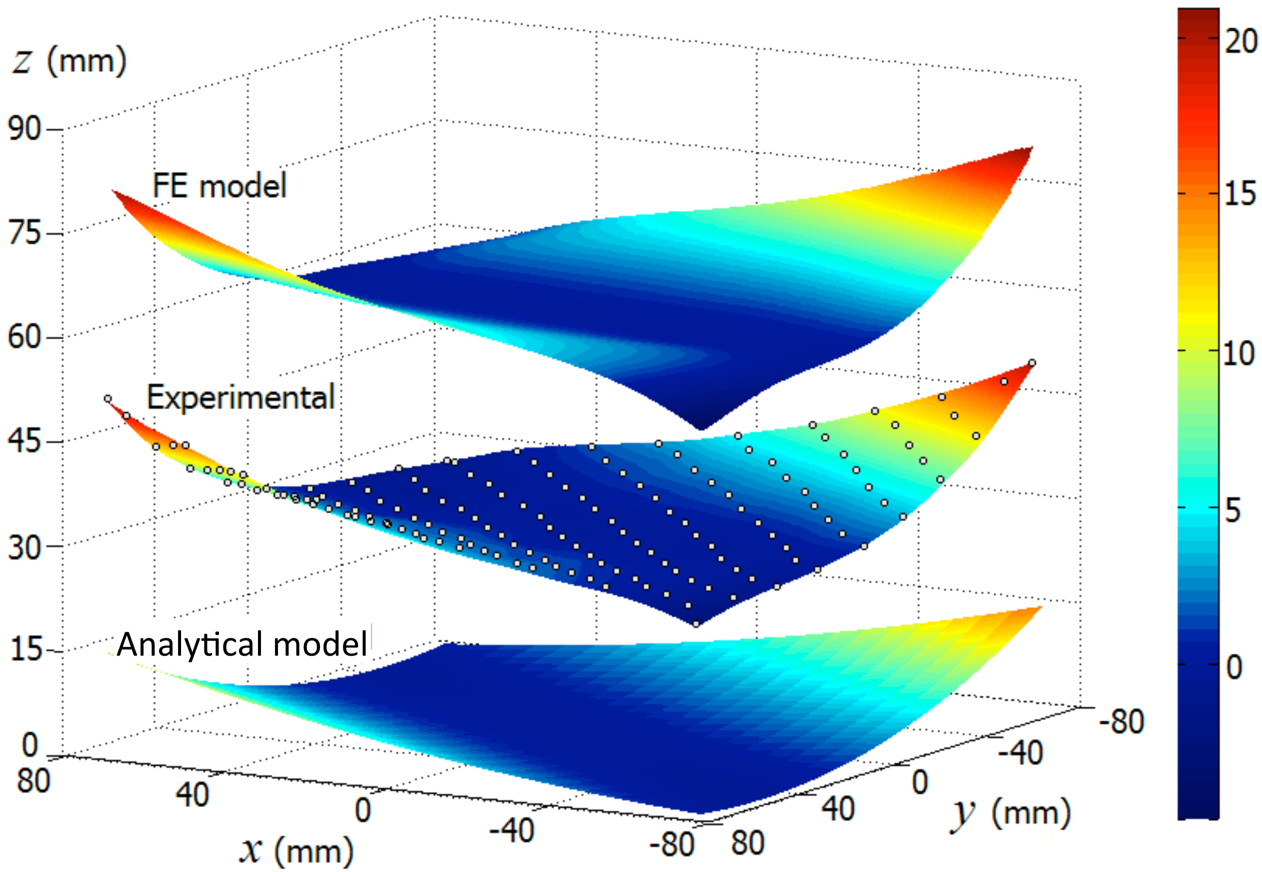
stiffness, thermal forces and moments,

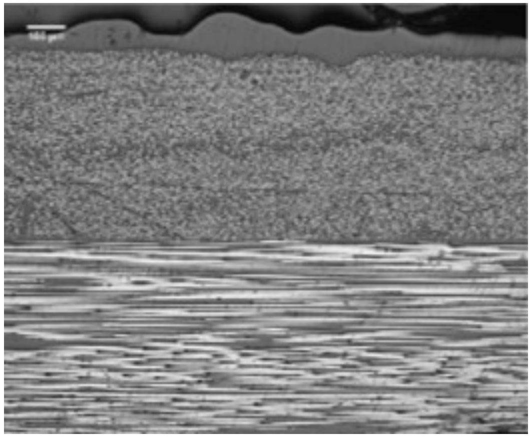
laminate geometry

$$f_i = \frac{\partial W}{\partial e_i} = 0; \quad i = 1 \ldots 14$$

where the e_i' s are the coefficients a, b, c, $d_{1\text{-}11}$.

[-30/60/0_{MFC}] laminate





- Laminate specific parameters
- Resign seepage
- Imperfections
- Limitation of the parabolic shape function
- Effects of MFC local stiffening

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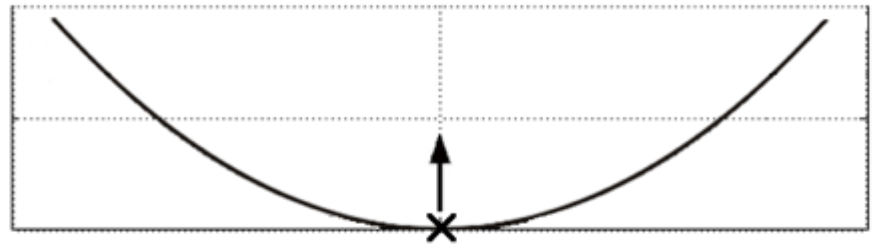
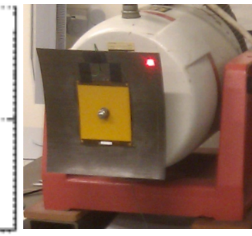
Modelling and investigation of dynamics

Dynamics Model

- Nonlinear dynamics
- Electro-mechanical

Base excitation





Analytical expression for nonlinear stiffness

Electro-mechanical coupling

$$\left\{\begin{array}{l}\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \boxed{\mathbf{K}(\mathbf{x})} + \theta\mathbf{V} = \mathbf{F}(t) \\ C_p\dot{\mathbf{V}} + \dfrac{\mathbf{V}}{R_l} + \theta\dot{\mathbf{x}} = 0\end{array}\right.$$

Betts, Kim, Bowen, Guyer, Le Bas, Inman, AIAA SciTech 2012

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Stiffness, K

out-of-plane displacement $$w = \frac{1}{2}\left(ax^2 + by^2\right)$$

Strain energy
$$W = \frac{1}{2}\int_{-L_y/2}^{L_y/2}\int_{-L_x/2}^{L_x/2}\left[\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{B} & \mathbf{D}\end{bmatrix}\begin{Bmatrix}\boldsymbol{\varepsilon}^0\\ \boldsymbol{\kappa}^0\end{Bmatrix} - 2\begin{Bmatrix}\mathbf{N}^{\mathrm{T}}\\ \mathbf{M}^{\mathrm{T}}\end{Bmatrix}\right]^T\begin{Bmatrix}\boldsymbol{\varepsilon}^0\\ \boldsymbol{\kappa}^0\end{Bmatrix}dxdy$$

Material stiffness properties

Residual thermal forces and moments

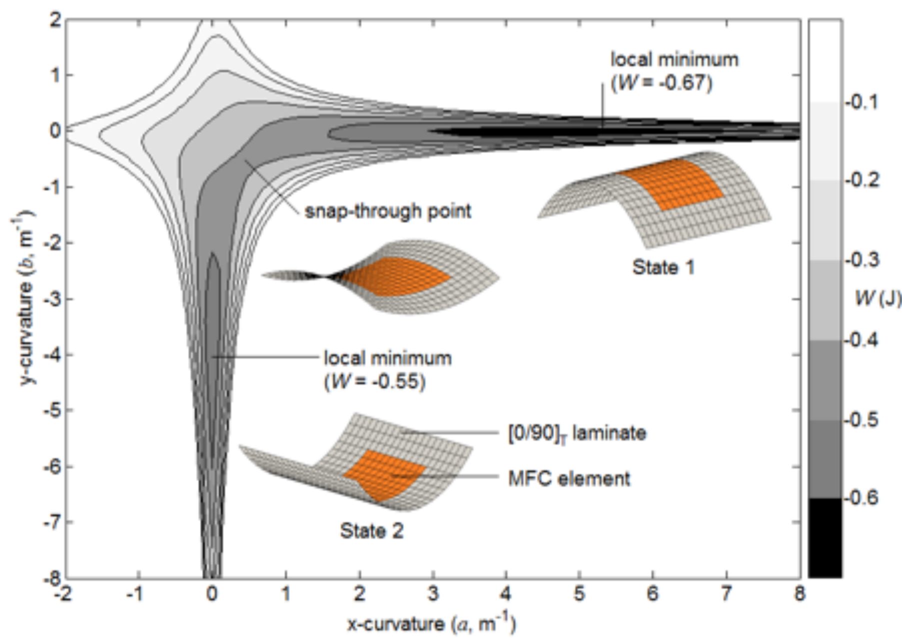
Strains and curvatures (functions of *a* and *b*)

which leads to

$$W = g_1 a^2 b^2 + g_2 ab + g_3 a^2 + g_4 b^2 + g_5 a + g_6 b + g_7$$

and $\mathbf{K} = \dfrac{\partial W(a,b)}{\partial \mathbf{x}}$

where $\mathbf{x} = [a, b]^{\mathrm{T}}$



Mass and Damping

$$\boxed{\mathbf{M}\ddot{\mathbf{x}}} + \boxed{\mathbf{D}\dot{\mathbf{x}}} + \mathbf{K}(\mathbf{x}) + \theta\mathbf{V} = \mathbf{F}(t)$$

$$C_p\dot{\mathbf{V}} + \frac{\mathbf{V}}{R_l} + \theta\dot{\mathbf{x}} = 0$$

- Mass

$$\mathbf{M} = \mathbf{M}_{\mathbf{lam}} + \mathbf{M}_{\mathbf{mfc}}$$

- Determined from work done by inertial forces

- Rayleigh proportional damping

$$\mathbf{D} = \alpha\mathbf{M}\dot{\mathbf{x}} + \beta\mathbf{K}(\mathbf{x})$$

α, determined experimentally by measuring the decay response of oscillations at the resonant frequency using a laser vibrometer.

β, assumed negligible as the mass term dominates for the relevant frequency range

Electrical and Coupling Terms

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}(\mathbf{x}) + \theta\mathbf{V} = \mathbf{F}(t)$$

$$C_p\dot{\mathbf{V}} + \frac{\mathbf{V}}{R_l} + \theta\dot{\mathbf{x}} = 0$$

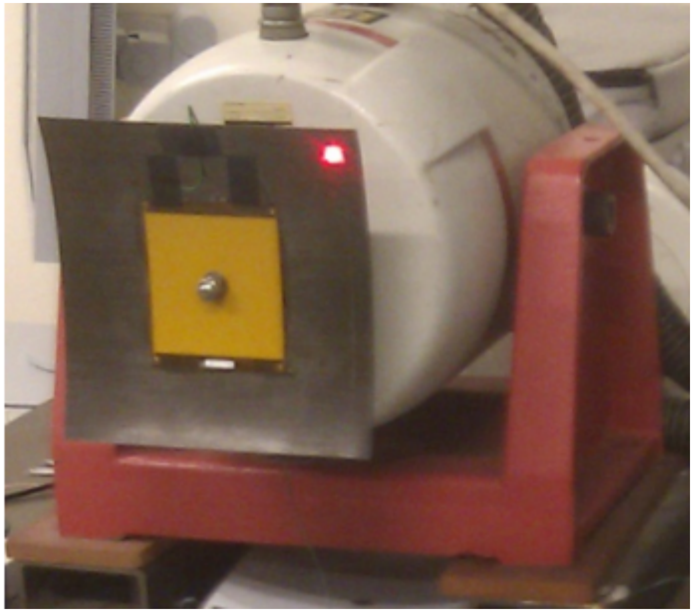
C_p Capacitance of the piezoelectric element (670nF)
Measured experimentally

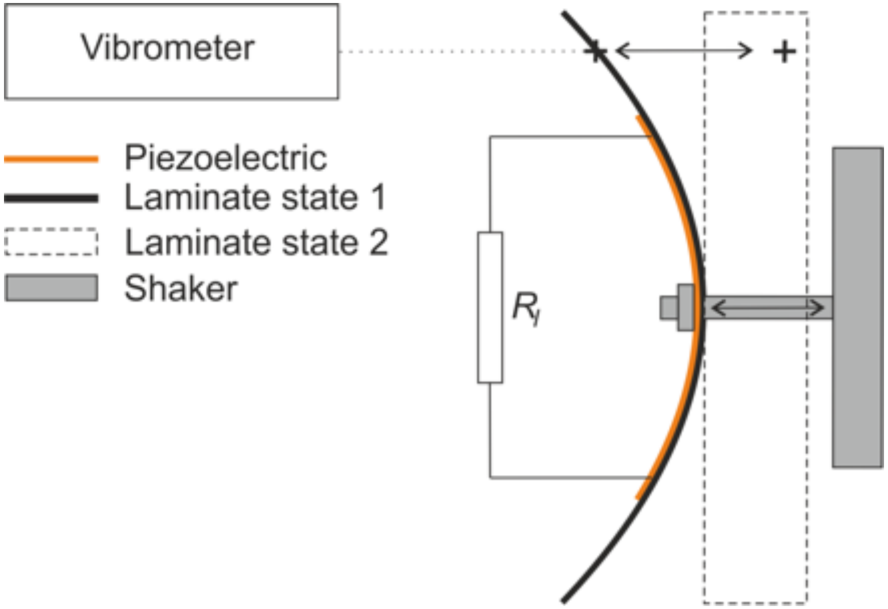
R_l Load resistance attached across the piezoelectric element (10kΩ)
Remains fixed throughout this work
The optimal value of R_l to maximise power output is frequency dependent. The value used here is not varied as the frequency changes, but is chosen to fit well within the frequency range considered

θ Piezoelectric electromechanical coupling coefficient (0.48)
Manufacturer's value

Experimental Set-Up

- 200 x 200 x 0.5mm laminate (HTA/913)
- $[0/90]_T$ stacking sequence
- 85 x 85 x 0.3mm MFC (P2-type)
- 10kΩ load resistance
- Centrally mounted to a electrodynamic shaker





System to reach a steady state – 30 seconds

Data recorded for the subsequent 6 seconds

Input: frequency and acceleration (g-level)

Output: Displacement and **Velocity** (laser vibrometer) and **Voltage** (oscilloscope)

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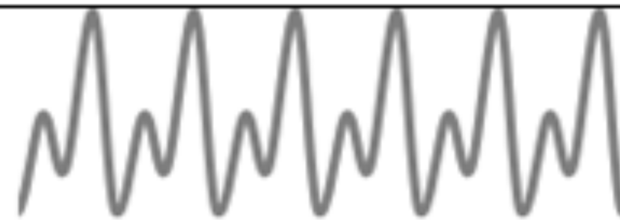
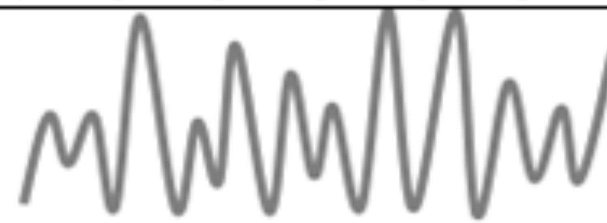
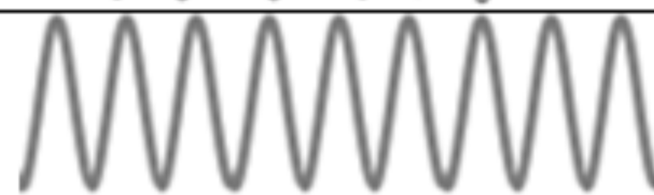
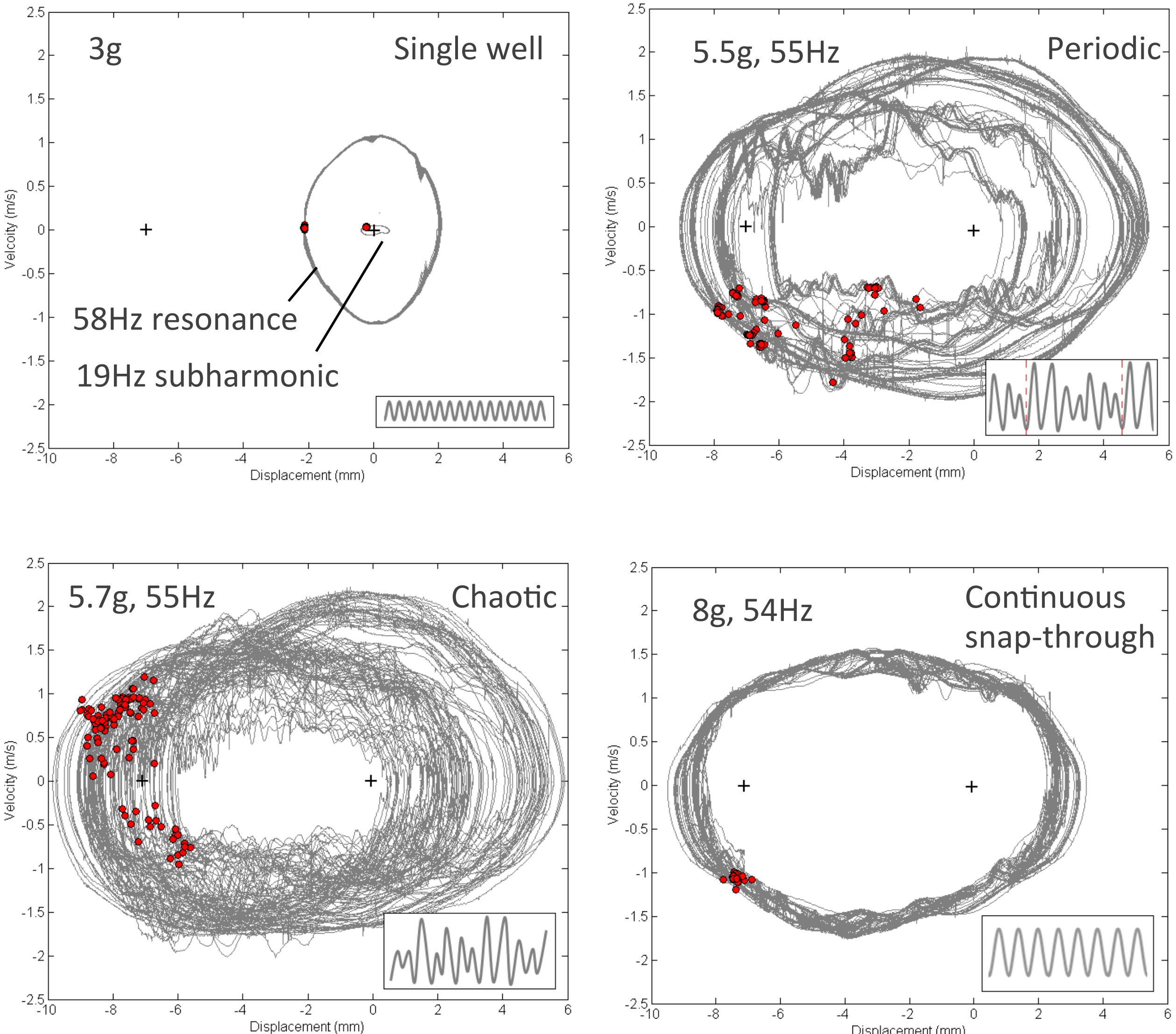
Characterisation of Dynamics Modes

Mode Type		**Example Waveform**
Single well oscillations	Resonance	
	Subharmonic	
	Off-resonance	
Intermittency	Periodic	
	Chaotic	
Continuous snap-through		

Investigation of Dynamics Modes [Experimental]



\+ stable states

start position of each time period

- Off-resonance single well not plotted (too small).
- A 7-cycle pattern is evident in the periodic solution
- Intermittent behaviour shows some higher peak velocities.
- Continuous snap-through shows high velocities (hence power) at every cycle.

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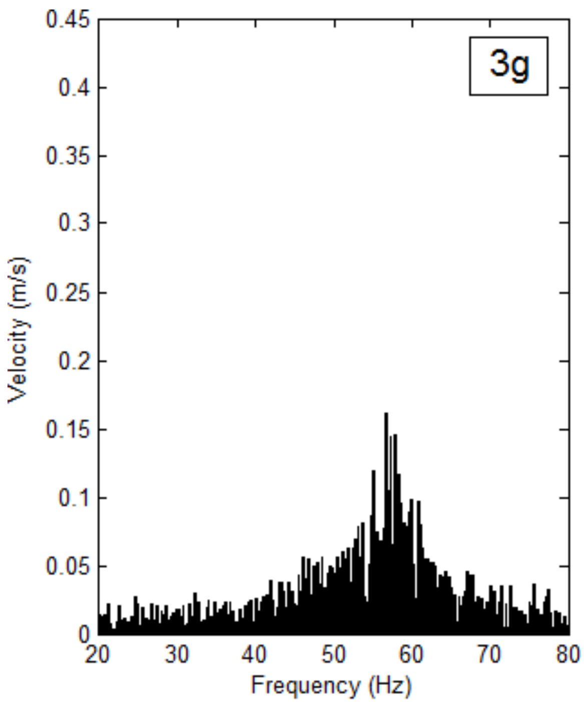
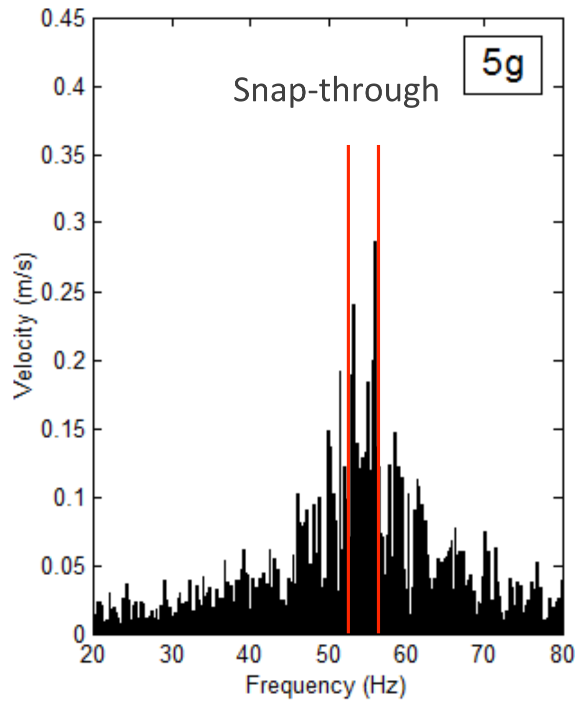
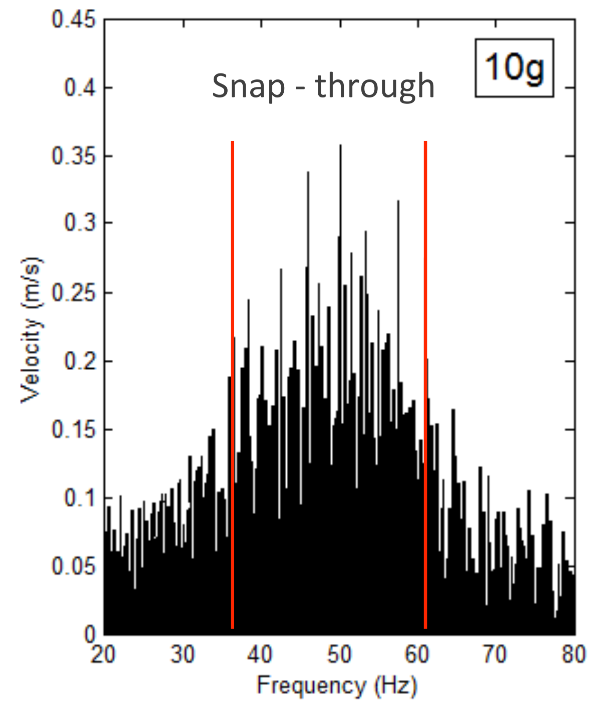
What conditions needed for continuous snap-through?



Linear – no snap-through



Narrow band of snap-through



Wide band of snap-through

FFT plots of corner velocity generated for slow frequency sweeps (0.1Hz/s) at three different amplitudes.

Snap-through is dependent on a combination of **drive frequency** and **amplitude**.

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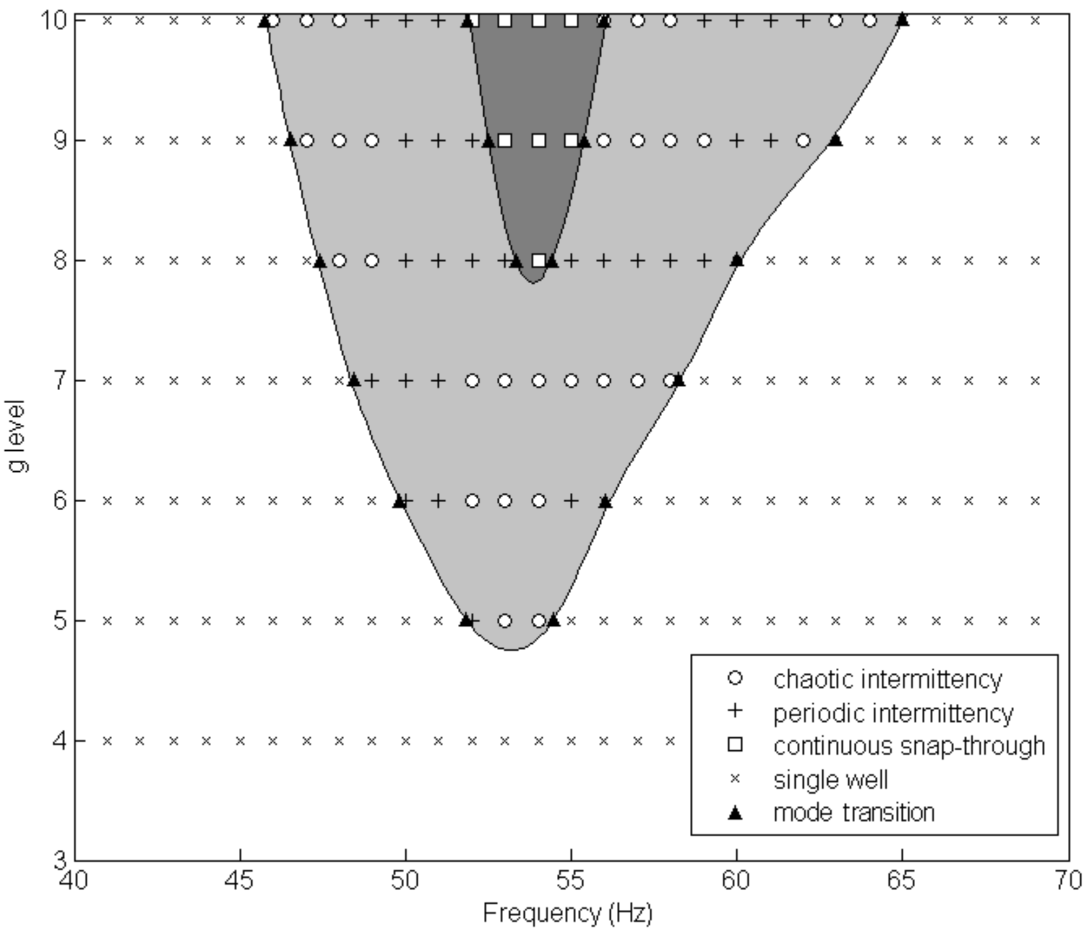
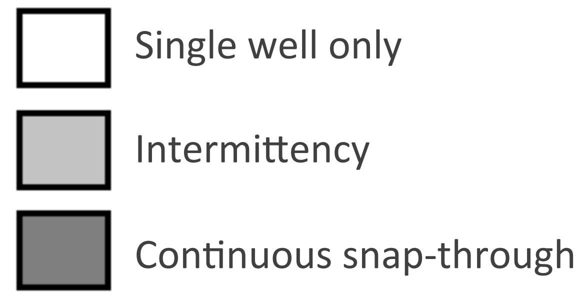
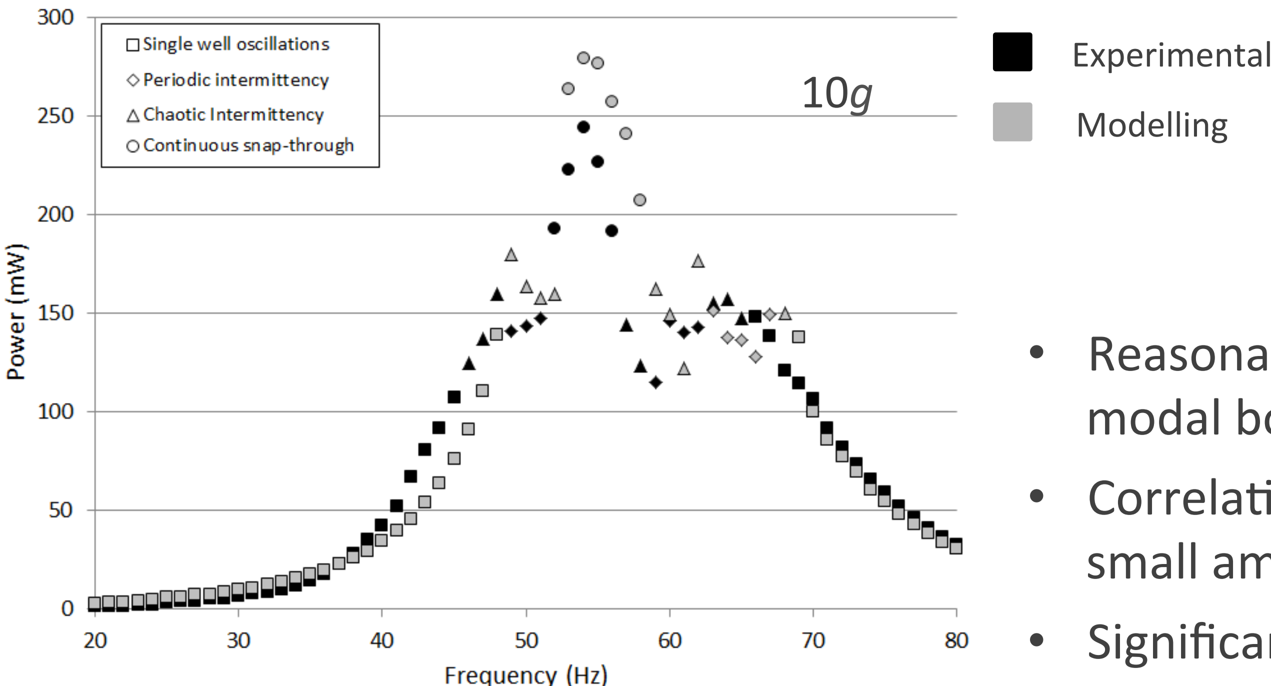
Mode Dependency on Vibration Characteristics





- Intermittency modes change repeatedly between periodic and chaotic.
- Broadband snap-through with increasing g-level.

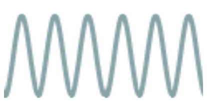
Power Output [Experimental & Modelling]




Single well oscillations 148mW (139mW)


Periodic intermittency 149mW (151mW)


Chaotic intermittency 160mW (180mW)


Continuous snap-through 244mW (279mW)

- Reasonably good at modelling the modal boundaries
- Correlation in power is good for small amplitude oscillations
- Significant discrepancies in power in the intermittent region.
- Highest power output in continuous snap-through at 54Hz (both modelling and experiments).

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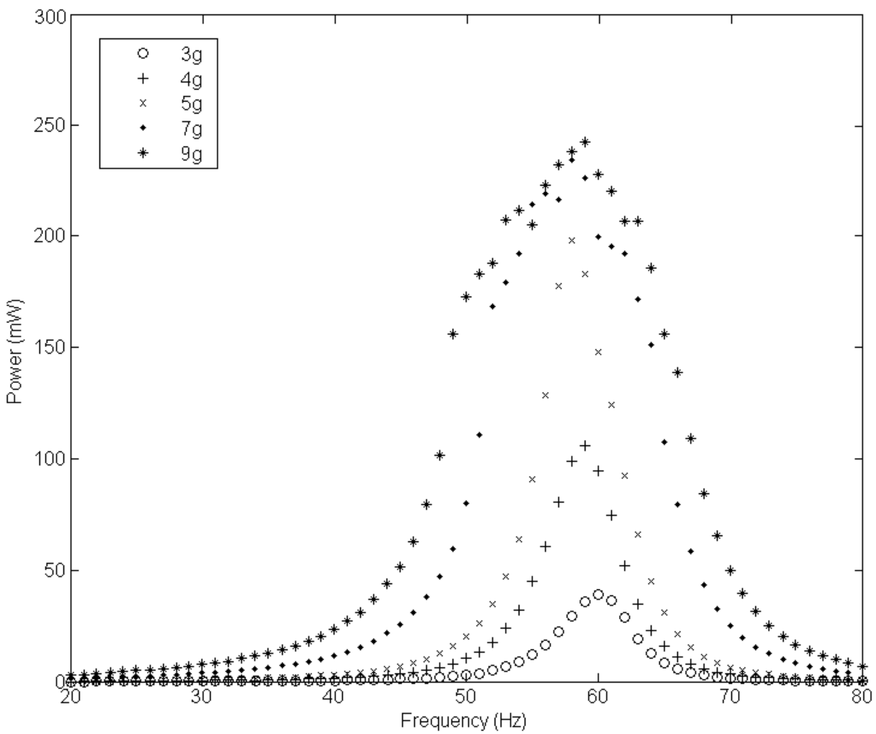
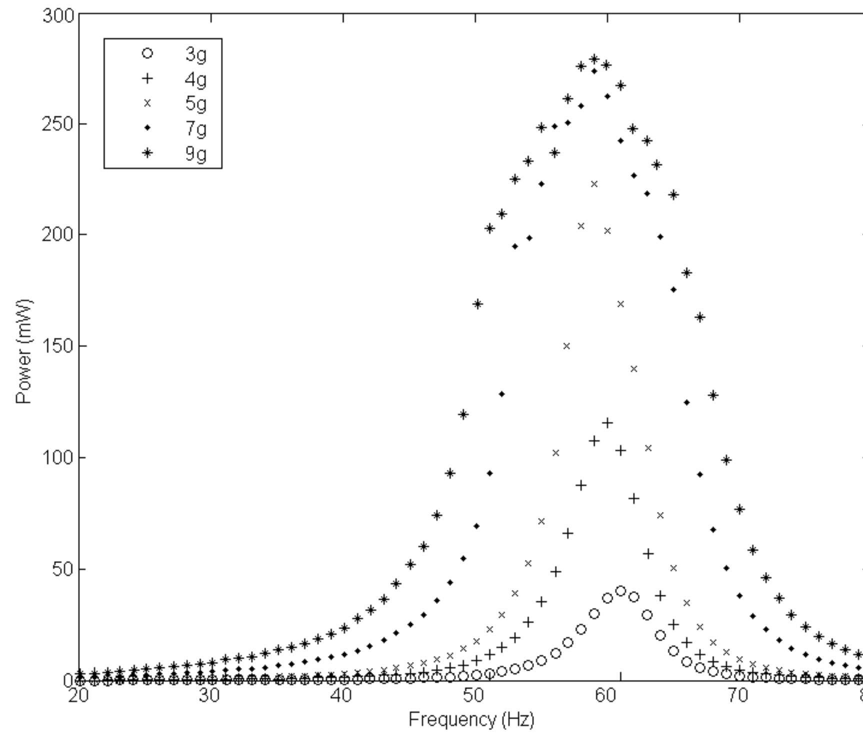
Average Power Outputs





- Snap-through first occurs at 5g.
- Peak power does not significantly increase for beyond 5g.
- Half-power bandwidth (frequency band over which half the peak power is observed) continues to increase (7Hz at 3g vs 22Hz 10g).
- Modelling consistently over-predicts power by ~15%.

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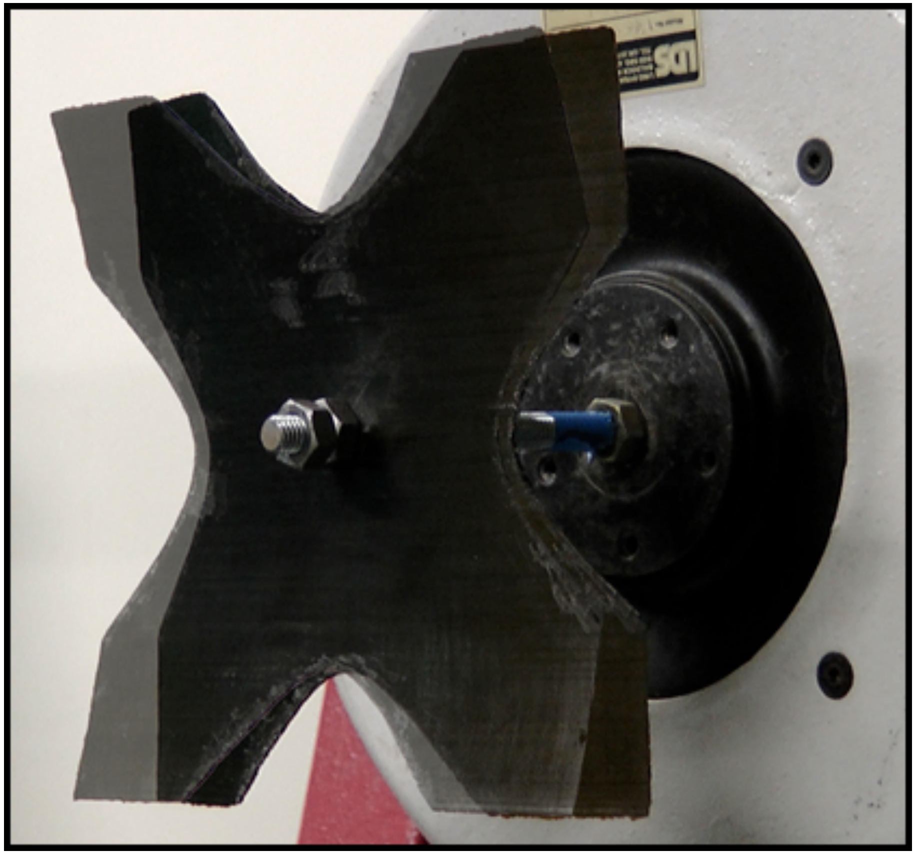
Arbitary shape



Generalise the existing modelling above to consider arbitrary planforms

Assume that static shapes of arbitrary geometry bistable laminates are still be approximated by the out-of-plane displacement profile

$$\text{out-of-plane displacement} = \frac{1}{2}(ax^2 + by^2 + cxy)$$

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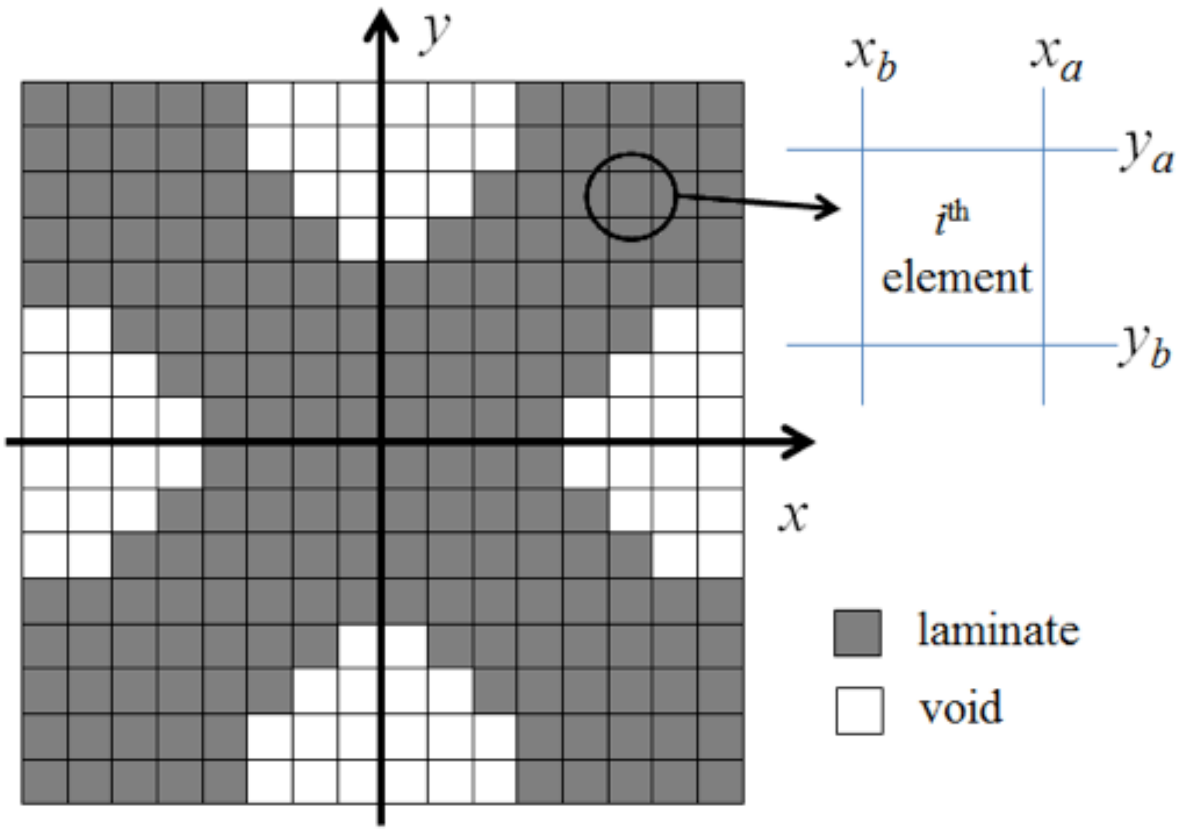
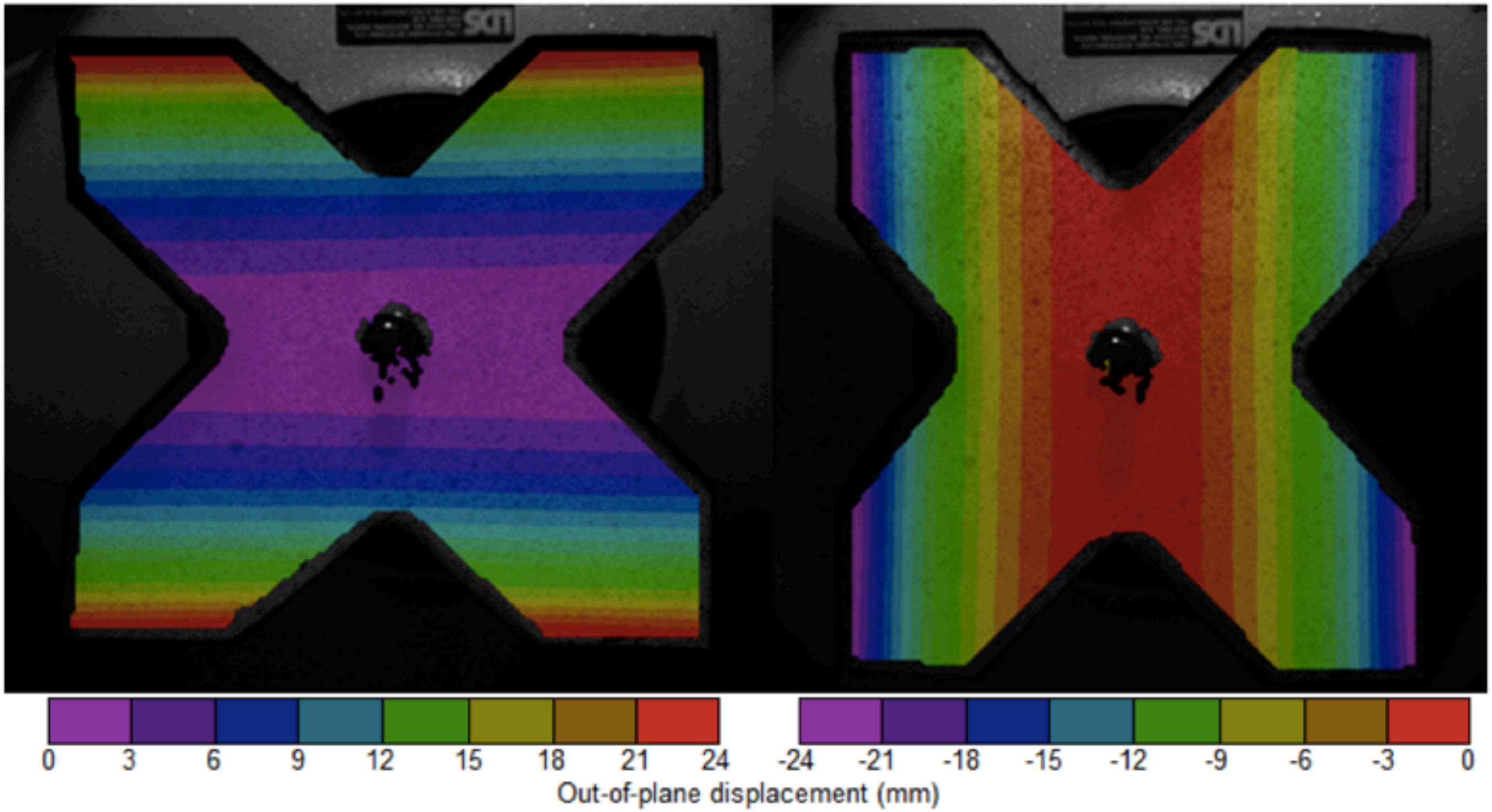
Arbitary shape



Discretisation of an example bistable laminate planform.

Arbitary static shape



Out-of-plane displacement of both stable shapes of the ×20 laminate. Red nearest the camera, purple furthest away.

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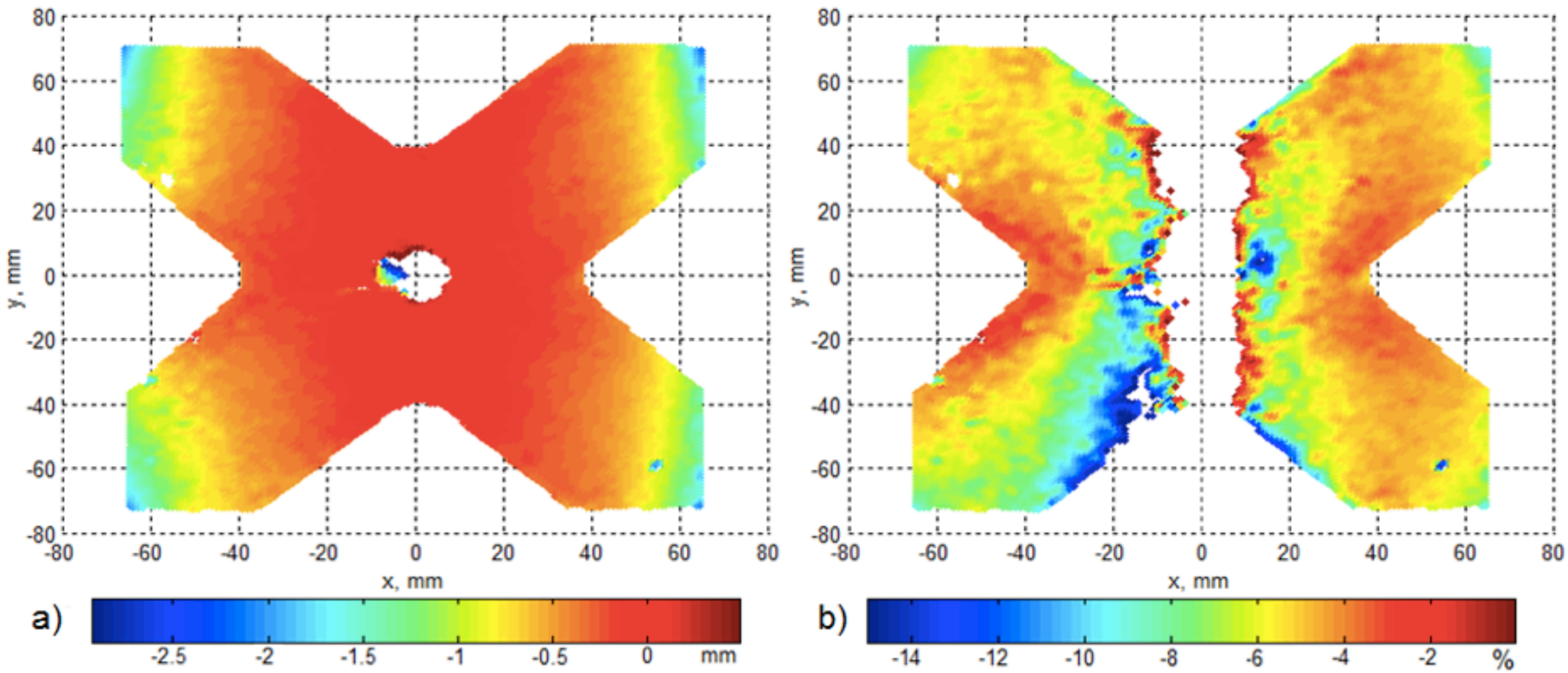
Model and experiment comparison



Difference between experimental and predicted shapes of ×20 laminate in mm and b) as a percentage of the measured value.

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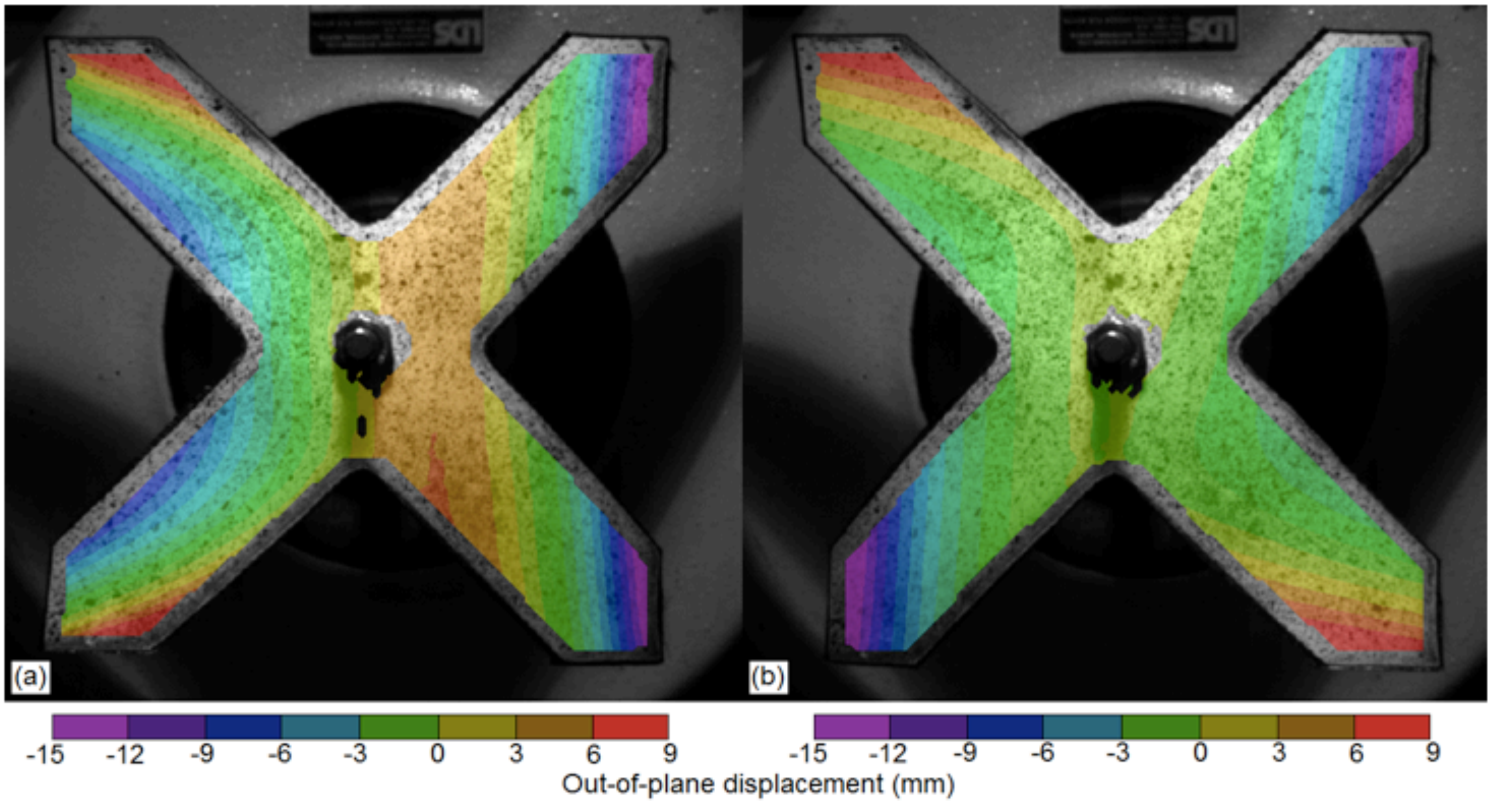
Limitations of model



Out-of-plane displacement of the ×40 laminate in two additional stable configurations (a) top left and bottom left sections are snapped forward and (b) top left and bottom right sections are snapped forward.

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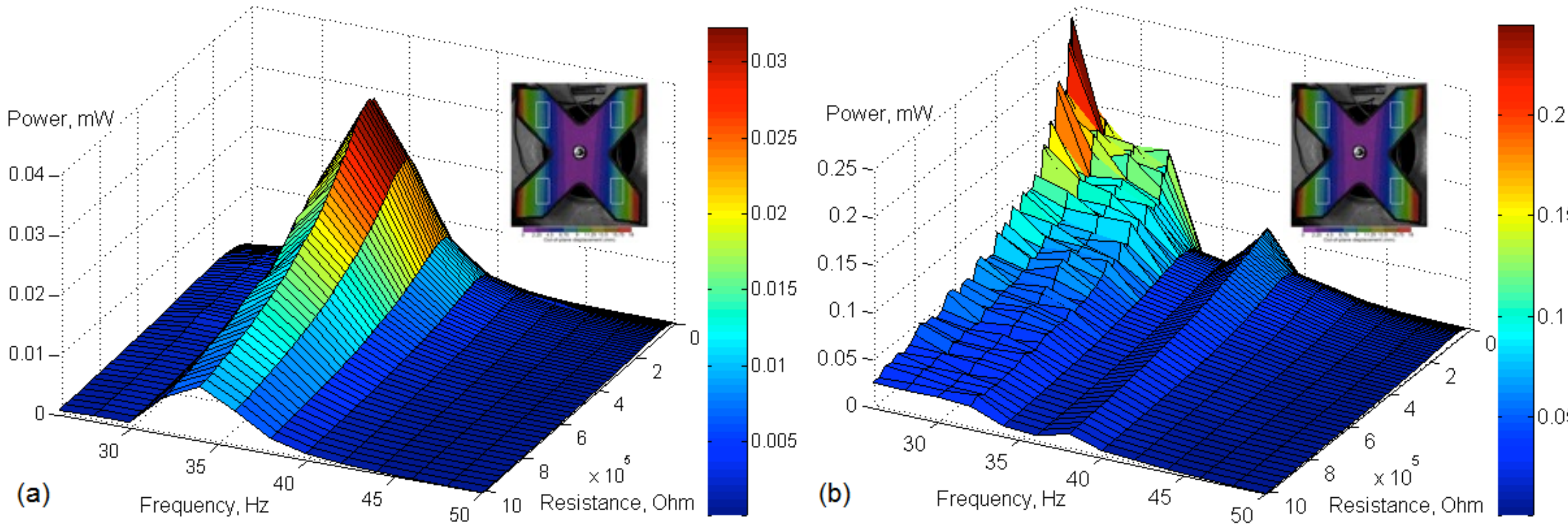
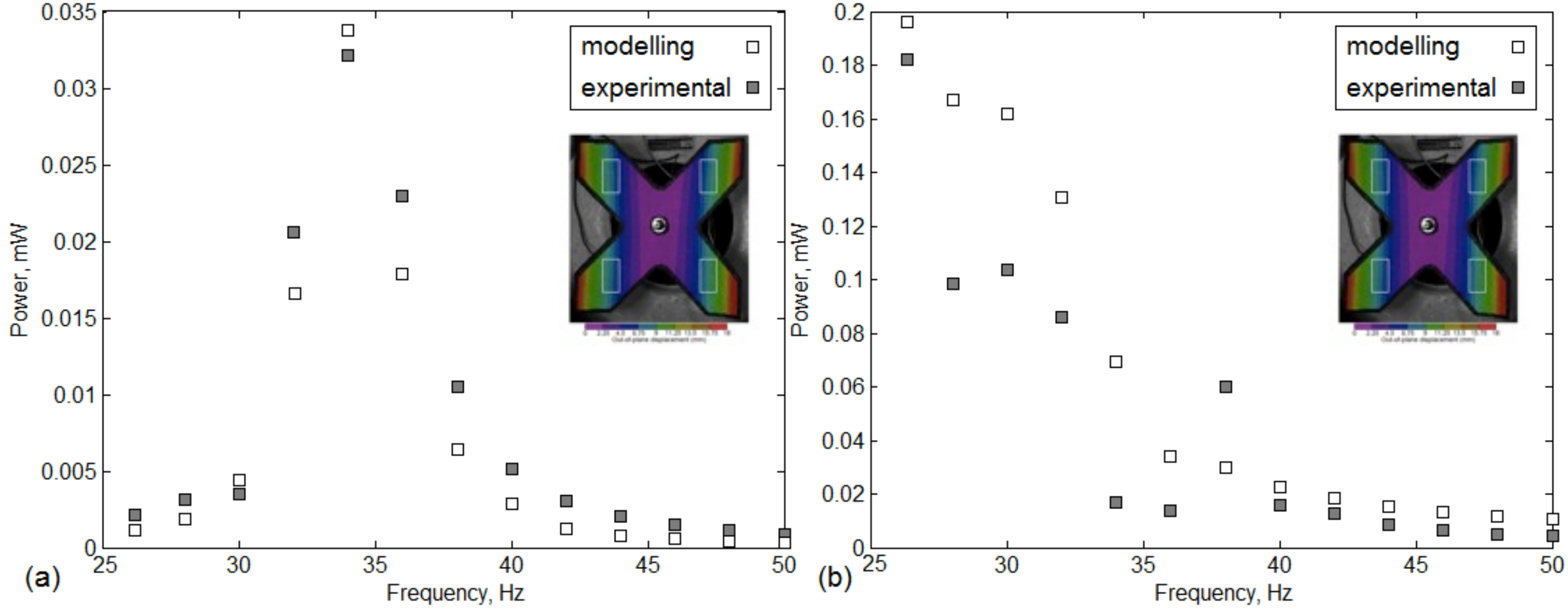
Addition of piezoelectrics (experimental with NPL)



Experimental power outputs for a single MFC (one of four) attached to ×30 laminate at (a) 3g peak acceleration and (b) 9g peak acceleration in the range 26-50Hz and 1kW-1MW.

Addition of piezoelectrics (experimental and model)



Experimental and modelling power outputs for a single MFC (one of four) attached to the surface of the ×30 laminate at (a) 3g peak acceleration and (b) 9g peak acceleration in the range 26-50Hz with resistance load of 124kW.

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Conclusions

- Analytical, FE and experimental investigation of bistable piezo-composites.
- Developed a dynamics model for base-excited bistable composite energy harvesters.
- Complex nonlinear dynamics modes identified.
- The model is reasonably good for predicting the modal boundaries and peak power.
- Highest power outputs in continuous snap-through modes – 244mW at 10g acceleration for a 200x200mm laminate with 85x85mm piezoelectric.
- Arbitary shape model – good agreement with static shape (5-8%) and power level
- Tool for optimisation of bistable laminates for energy harvesting

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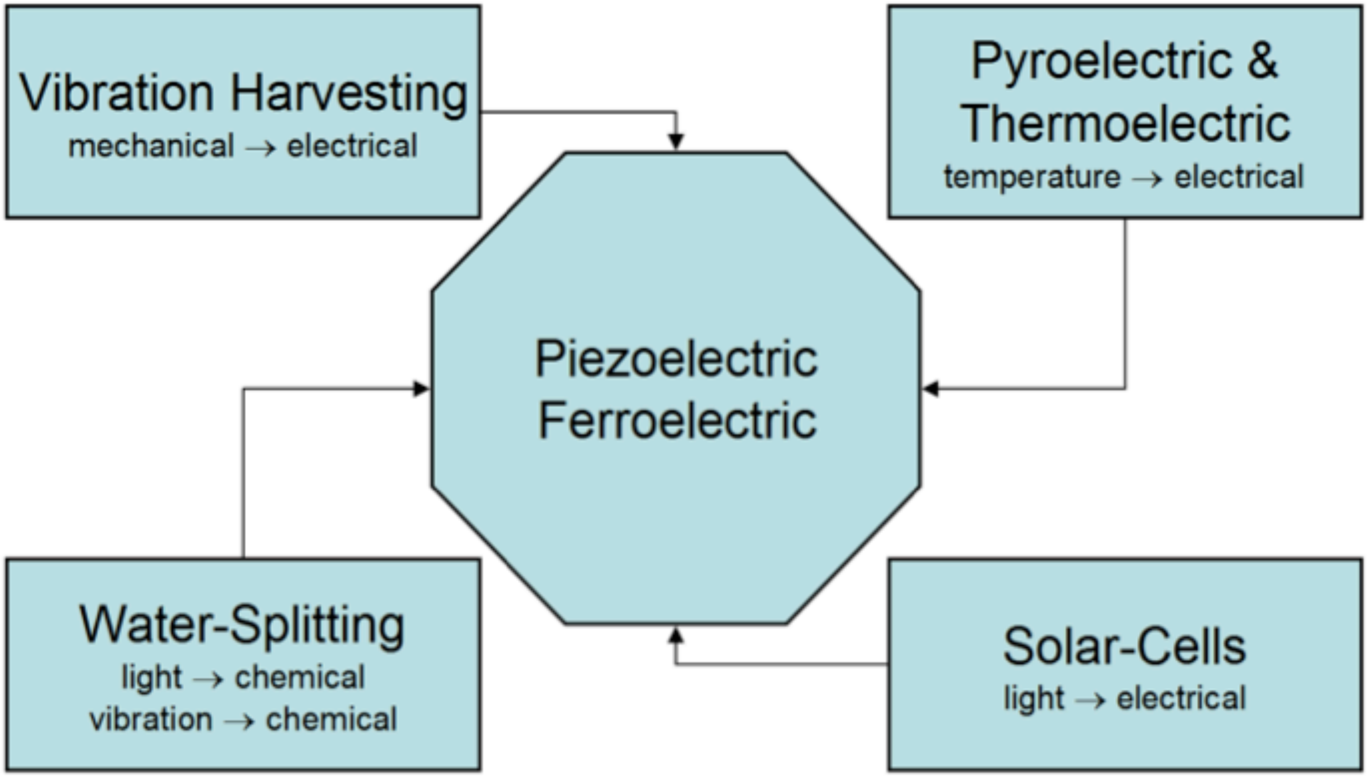
Novel Energy Materials: Engineering Science and Integrated Systems (NEMESIS)

ERC Grant Agreement no. 320963
Prof. Chris Bowen



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