Optimization of Piezoelectric Bistable Composite Plates for Broadband Vibrational Energy Harvesting

UNIVERSITY OF BATH

David N. Betts
H. Alicia Kim
Christopher R. Bowen
Daniel J. Inman

Introduction

We present an arrangement of bistable composite plates[1] with bonded piezoelectric patches (Figure. 1) to perform broadband vibration-based energy harvesting from ambient mechanical vibrations. This offers the following advantages:

1. improved power generation compared to conventional resonant systems,
2. reduced device complexity due to the inherent structural bistability,
3. can occupy smaller volumes than bistable magnetic cantilever systems.

This study aims to maximize the energy generation by discovering the correct geometric configuration for harvesting. We consider the optimal device aspect ratio, thickness, stacking sequence, and piezoelectric size. Increased electrical energy output is found for geometries and piezoelectric configurations which have not been considered previously.

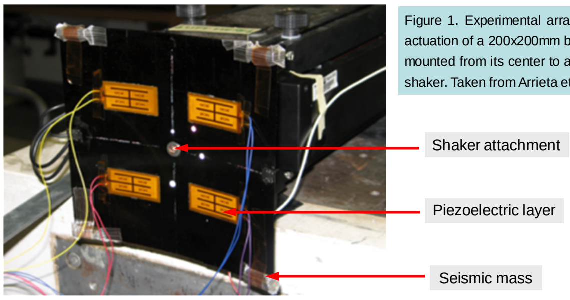

Figure 1. Experimental arrangement for actuation of a 200x200mm bistable plate mounted from its center to a mechanical shaker. Taken from Arrieta et al.[1]

Bistable composite plates

Asymmetric layup laminates have differing thermal properties between plies leading to residual strains which induce a curved deformation[2]. Under certain geometric conditions the thermal strain can lead to two stable shapes (Figure. 2). 'Snap-through' between these shapes results in large deflections and energy generation.

Bistable piezo-composites (Fig. 3) have been extensively studied for morphing / adaptive structure concepts[3-5]. Here we consider their use for broadband energy harvesting of ambient mechanical vibrations.

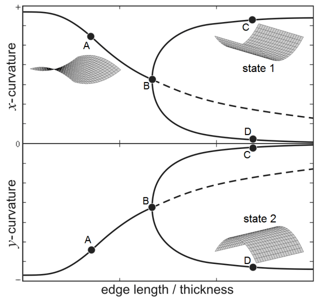

Figure 2. Stable (solid line) and unstable (dashed line) shapes of $[0/90]_T$ laminates with variation in the geometry.

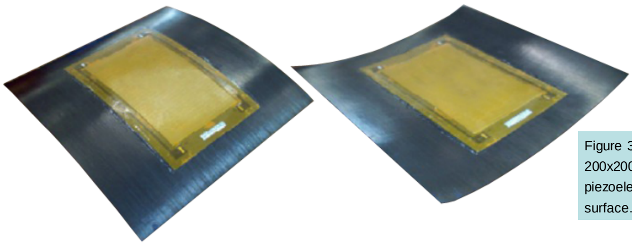

Figure 3. Two stable cylindrical shapes of a 200x200mm bistable plate $[0/90]_T$ with a piezoelectric MFC patch attached to one surface.

Device Configuration

The device configuration is based on a previous experimental study[1], and shown in Figure. 4. A mechanical actuator is attached to the centre point such that z-displacement induces snap-through. Four piezo-elements on each surface are used to harvest energy from the alternating stress excitation.

Laminate shapes are calculated using an analytical model[4], incorporating the piezoelectric configurations.

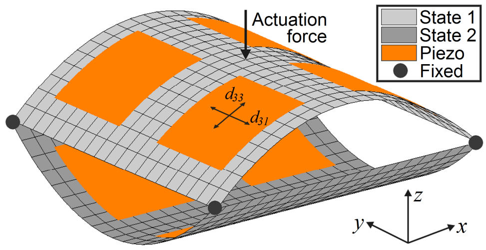

Figure 4. Actuation arrangement for a $[0^P/0_2/90_2/90^P]_T$ laminate, with 40% piezo coverage. P denotes piezo-elements with 0° or 90° poling direction.

We aim to maximize the electrical energy U generated by a bistable piezo-composite structure. We model this based on the statics of the system. When operating off-resonance a piezoelectric layer behaves as a parallel plate capacitor. Hence the electrical energy generated is given by Equation. 1, where d is the effective piezo-electric strain constant, g is the effective piezoelectric voltage constant, σ is the stress, A is the surface area of the layer, and t_p its thickness.

$$U = \frac{1}{2}(dg)\cdot\sigma^2\cdot(At_p) \qquad (1)$$

Results

The optimal designs are determined by a sequential quadratic programming routine, limiting strain to below failure levels and ensuring bistability. Two solutions are in Table 1.

- The global optimum has ideal piezoelectric alignment and large laminate curvature.
- The local optimum has ideal piezoelectric alignment and reduced curvature.
- The maximum strain constraint is found to be inactive for both designs.
- The thickness does not correspond to maximum curvature, not an intuitive result.
- Optimum designs both have square aspect ratio
- The piezoelectric sizes differ greatly suggesting a balance between large curvature and and a large area from which energy can be harvested, see Figure 5.

Table 1. Local and global optimum solutions, limited below failure strain.

	Local Optimum	Global Optimum
Stacking sequence	$[0^P/90/0/90^P]_T$	$[0^P/0/90/90^P]_T$
Single ply thickness, mm	0.619	0.626
Piezoelectric area, %	42.40	72.43
Aspect ratio	1.0	1.0
Maximum strain, μstrain	1113	1097
Actuation force, N	3.39	3.41
Electrical energy, mJ	20.4	33.7

Design Parameter Studies

The optimum solutions are further investigated in a series of design parameter studies. The aim is to provide a better understanding of the complex interactions of the physical constraints and design requirements, and to illustrate the pattern of results around the optimum solutions.

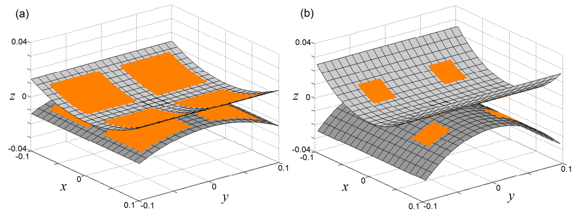

Figure 5. Shapes of $[0^P/0/90/90^P]_T$ laminates with (a) 50% and (b) 10% piezoelectric coverage.

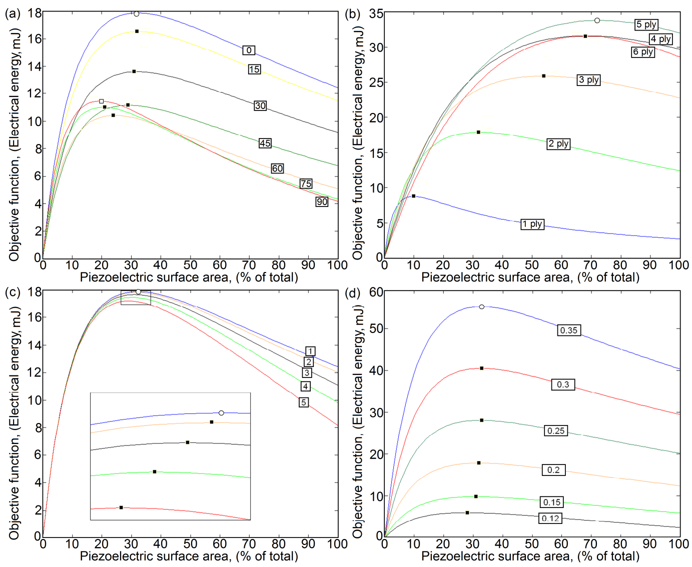

Figure 6. Change in electrical energy with piezo area and (a) θ (shown on each line) for $[0^P/\theta/\theta+90/90^P]_T$ layups, (b) n x 0.125mm plies (shown on each line) for $[0^P/0_n/90_n/90^P]_T$ layups, (c) aspect ratio (shown on each line) for $[0^P/0_2/90_2/90^P]_T$ layups, and (d) edge length (shown in m on each line) for $[0^P/0_2/90_2/90^P]_T$ layups. Black squares mark optima for each discrete variable value, white circles mark global optima, white squares mark local optima.

Layup – Figure. 6a shows change in electrical energy U with layup for $[0^P/\theta/\theta+90/90^P]_T$ laminates. Optima for each θ value form a convex hull which helps to understand the local optimum. The global solution is found to be θ = 0°.

Thickness – Figure. 6b shows change in U with number of plies n for $[0^P/0_n/90_n/90^P]_T$ laminates. As thickness increases, the electrical energy generated increases to an optimum of n = 5. Beyond this point, the laminate approaches the loss of bistability (see Figure. 1) and energy quickly reduces to zero.

Aspect ratio – Figure. 6c shows change in U with aspect ratio for $[0^P/0_2/90_2/90^P]_T$ laminates. A high aspect ratio laminate exhibits significantly larger deflection in one shape than the other, see Figure 7. However, while a square aspect ratio is optimal, variation in electrical energy with aspect ratio is small.

Surface area – Figure 6d shows variation in U with surface area (fixed previously) for $[0^P/0/90/90^P]_T$ laminates. It is found that as the edge length of the device increases, the electrical energy generated increases approximately linearly, making the problem scalable.

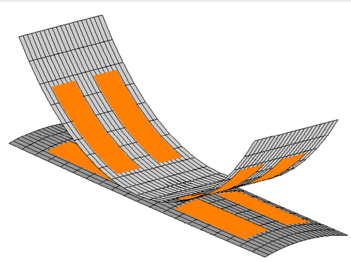

Figure 7. Shapes of a $[0^P/0/90/90^P]_T$ laminate with an aspect ratio 3 and 40% piezoelectric coverage.

Conclusions and Future Studies

This optimization study has identified the correct geometric configurations for high electrical energy generation in bistable piezo-composite harvesting devices, by considering the statics of the device. Optimal designs are found to have:

Square aspect ratio, cross-ply layups, piezoelectric alignment with major curvature, maximum thickness without compromising the bistable characteristics, and a balance of piezoelectric area with all of the above factors.

Where the device may experience multiple frequencies, there may be a balance between reasonable energy harvesting across a vibration pattern, and tuning for a single frequency. Future work will therefore focus on the dynamics of the device.

1. Arrieta, A.F., Hagedorn, P., Erturk, A., and Inman, D.J., "A piezoelectric bistable plate for nonlinear broadband energy harvesting," *Applied Physics Letters*, 97(10), pp. 1-3, (2010).
2. Hyer, M.W., "Calculations of the room-temperature shapes of unsymmetric laminates," *Journal of Composite Materials*, 15(July), pp. 296-310, (1981).
3. Betts, D.N., Salo, A.I.T., Bowen, C.R., and Kim, H.A., "Characterisation and modelling of the cured shapes of arbitrary layup bistable composite laminates," *Composite Structures*, 92(7), pp. 1694-1700, (2010).
4. Dano, M.L., and Hyer, M.W., "Thermally induced deformation behavior of unsymmetric laminates," *International Journal of Solids and Structures*, 35(17), pp. 2101-2120, (1998).
5. Betts, D.N., Kim, H.A., and Bowen, C.R., "Optimization of stiffness characteristics for the design of bistable laminates," *AIAA Journal*, Accepted, (2012).
6. Betts, D.N., Kim, H.A., and Bowen, C.R., "Modeling and optimization of bistable composite laminates for piezoelectric actuation," *Journal of Intelligent Material Systems and Structures*, 22(18), pp. 2181-2191, (2011).

EPSRC
Pioneering research and skills
Grant number EP/J014389/1