Nonlinear Considerations in Energy Harvesting

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Two key issues in energy harvesting can be solved by introducing nonlinear effects.

For identical inputs to a linear system and a nonlinear system, etc., a drastically improved voltage results which is broadband and of larger magnitude.

...
There are many conventional ways of making broadband energy harvesters...

Voltage FRFs (broadband configuration)

Piezoelectric bimorphs

Electromagnetic shaker

Acceleration

Voltage FRFs (single cantilever)

Comparison of the voltage FRFs

Not an outstanding design in terms of the power density...

Others use electromagnetic transduction (Beeby, et al)
Hardening stiffness of the monostable Duffing oscillator has been investigated by others to increase the bandwidth of operation.

Piezoelectric energy harvester with cubic stiffness

\[ \ddot{x} + 2\varepsilon\omega_n\dot{x} + \omega_n^2 x + \varepsilon\alpha x^3 - \varepsilon\chi v = \varepsilon f \cos \omega t \]

\[ \dot{v} + \lambda v + \kappa \dot{x} = 0 \]

The high-energy branch can be lost due to the shunt damping effect of the electrical load (weak nonlinearity).
Here We Examine Using a Bistable Piezomagnetoelastic Beam

(Moon and Holmes, 1979)

Magnets added near the tip of a cantilever introduce nonlinearity
Limit Cycle Oscillations for Broad Band Harvesting

- A magnetic field causes the equation of motion of the harvesting piezoelectric cantilever to be nonlinear

\[ \ddot{x} + 2\zeta \dot{x} - \frac{1}{2} x (1 - x^2) - \chi v = f \cos \Omega t \]

- Spacing of the magnets results in:
  - 5 equilibrium (3 stable)
  - 3 equilibrium (2 stable)
  - 1 equilibrium (1 stable)

- Limit cycle oscillation is the possible producing large amplitude periodic response over a range of input frequencies
The piezomagnetoelastic energy harvester configuration has been investigated theoretically and experimentally.

First the *strange attractor* (Moon and Holmes, 1979) is captured in the chaotic response of the piezomagnetoelastic configuration.
Large-amplitude periodic response is obtained by changing the forcing level or the initial conditions.

(1) Transient chaos followed by high-energy limit cycle oscillations (large-amplitude periodic attractor)

0.57g (RMS) input at 8 Hz

(2) Co-existing attractors (strange attractor and large-amplitude periodic attractor)

0.35g (RMS) input at 8 Hz

Theoretical simulations show the presence of these high-energy orbits at several frequencies.

\[
\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x(1-x^2) - \chi v = f \cos \Omega t
\]
\[
\dot{v} + \lambda v + \kappa \dot{x} = 0
\]

\[
\zeta = 0.01, \quad \chi = 0.05, \quad \kappa = 0.5 \quad \text{and} \quad \lambda = 0.05 \quad x(0) = 1, \quad \dot{x}(0) = 1.3, \quad v(0) = 0, \quad f = 0.08
\]
Experimental verification of the broadband high-energy orbits in the piezomagnetoelastic configuration

0.35g (RMS) at 8 Hz

0.35g (RMS) at 6 Hz
Large-amplitude response of the piezomagnetoelastic energy harvester yields an order of magnitude larger power output over a range of frequencies.

*Base acceleration (input)*

*Open-circuit voltage (output)*

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**Power**

- **5 Hz**
  - Average power (mW)
  - Load resistance [Ω]

- **6 Hz**
  - Average power (mW)
  - Load resistance [Ω]

- **7 Hz**
  - Average power (mW)
  - Load resistance [Ω]

- **8 Hz**
  - Average power (mW)
  - Load resistance [Ω]
Power Output Comparison of Linear vs Nonlinear

<table>
<thead>
<tr>
<th>Excitation Frequency</th>
<th>5 Hz</th>
<th>6 Hz</th>
<th>7 Hz</th>
<th>8 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezo-Magneto-Elastic</td>
<td>1.57 mW</td>
<td>2.33 mW</td>
<td>3.54 mW</td>
<td>8.54 mW</td>
</tr>
<tr>
<td>Piezo-elastic</td>
<td>0.10 mW</td>
<td>0.31 mW</td>
<td>8.23 mW</td>
<td>0.46 mW</td>
</tr>
</tbody>
</table>

Linear Resonance

Note that *at linear resonance* the linear system will always win, however it is narrow band and falls off quickly away from resonance and that the nonlinear has higher values overall.
Bistable piezo-carbon-fiber-epoxy plate

Courtesy of the Bristol Composites Group
A bistable carbon-fiber-epoxy plate exhibits similar nonlinear dynamics (no external magnets required).

The plate is clamped to a seismic shaker from its center point.

Linear FRFs around each stable state

The stable equilibrium positions are not symmetric with respect to the unstable one.

Various nonlinear phenomena can be observed in the bistable plate configuration.

**High-energy LCO (8.6 Hz)**

**Chaos (12.5 Hz)**

**Intermittent chaos (9.8 Hz)**

**Subharmonic resonance (20.2 Hz)**
Various nonlinear phenomena can be observed in the bistable plate configuration.

**Forward sweep**

**Backward sweep**

**Power curves**

Various nonlinear phenomena can be observed in the bistable plate configuration. For example, large-amplitude LCO, intermittent chaos, persistent chaos, and subharmonic resonance can occur. The power curves show the average power and excitation frequency for different phenomena.
Large-amplitude oscillations generate very high power output over a range of frequencies.

**Average power vs. Load resistance**

![Graph showing average power vs. load resistance with various curves labeled: Large amplitude LCO, Intermittency, Chaos, Subharmonic mode $\omega_{1}^{s1}$.]

**Average power vs. Frequency**

![Graph showing average power vs. frequency with a peak at 98.5 kohm.]
Nonlinear Hybrid Harvester

Piezoelectric Harvesting:
High voltage low current

Electromagnetic Harvesting
High current low voltage

Piezo element
Tip magnet
Coil

R1
R2
Mono-Stable : Magnet Spacing

Tip Velocity

PZT Power

Electro/Mag Power

R₁=100kΩ, R₂=5Ω, aᵦ=1.47, legend: δ (mm)

Tip vel/ Base accel (1/S)

PZT power/ Base accel² (Kg.S)

E.M. power/ Base accel² (Kg.S)
Bi-Stable: Base Acceleration

PZT Power

Electro/Mag Power

Tip Velocity
Acceleration data of the bridge has been simplified to a harmonic function for simulations in the lab.

3-span steel girder bridge (08/18/09 - Roanoke)

acceleration measured on the bridge

approximation as a persistent single harmonic (0.05g at 7.7 Hz)

Experimental setup

accelerometer

Piezoelectric and electromagnetic generators

Seismic shaker

acceleration measured on the shaker
Piezoelectric and electromagnetic power outputs have been measured for an acceleration input of $0.05g$ (RMS: $0.035g$) at 7.7 Hz.

Piezoceramic patches
Accelerometer
Seismic shaker

**Combined piezoelectric-electromagnetic generator configuration**

Electromagnetic part: $0.22$ V for $82$ ohms = **0.6 mW** (per coil)

Piezoelectric part: $11.2$ V for $470$ kohms = **0.3 mW**

**Power output of a single generator (for $0.05g$) = 0.9 mW**
Increased base acceleration amplitude results in a larger power output. (0.1\(g\), RMS: 0.07\(g\) at 7.7 Hz yields 2.7 mW).

**Electromagnetic part:** 0.42 V for 100ohms = 1.8 mW (from a single coil)

**Piezoelectric part:** 21 V for 470 kohms = 0.94 mW

**Power output of a single generator (for 0.1g) =** 2.7 mW
Is harvesting of flow through wing vibration possible?

At 9.23 m/s, 10.7 mW harvested AND
The corresponding shunt effect increased the flutter speed by 5.5%

Total damping vs. Airflow speed

Just a linear analysis for now but LCO does occur in aircraft
Summary and conclusions

- Bistable beam and plate configurations have been discussed for broadband energy harvesting.

- The beam configuration requires magnets for bistability whereas the plate configuration is bistable due to the laminate characteristics.

- The design problem is to achieve persistent snap through and nonlinear phenomena for the given excitation amplitude and frequency range.

- Combined E/M and PZT is promising for charging batteries
Some Funded Projects In the US

- Center for Energy Harvesting Materials and Systems (National Science Foundation/Industry program)
  - ITT: new lead free piezoelectric materials
  - UTRC: building applications (running infrastructure sensors)
  - SAIC: submerged river sensors (flow harvesting)
  - Texas Instruments: TBA
  - Physical Acoustics Corporation: Harvesting for running AE sensors
  - Texas MicroPower: MEMs Zigzag harvester
- National Institute for Standards and Testing
  - 50 million USD in harvesting and monitoring of Bridges
- Air Force Office of Scientific Research
  - 6 million USD for harvesting in UAVs
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- F 9550-09-1-0625: “Simultaneous Vibration Suppression and Energy Harvesting”

Shameless, Self Serving References


National Science Foundation Center for Energy Harvesting Materials and Systems

http://cehms.mse.vt.edu

Thank you!.. Questions?